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Flux tube structure of dual QCD and confinement

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Abstract : Study of a dynamical model for analyzing the mechanism of color confinement in QCD has been undertaken and the flux tube structure of the resulting dual gauge theory has been investigated. Using the magnetic symmetry of QCD and the fibre bundle formulation of dual QCD, the topological charges have been shown to lead a unique dual dynamics in QCD, which has its immediate implications in non-perturbative regime. Such dynamics is then shown to derive the magnetic condensation in QCD vacuum by the dynamical breaking of magnetic symmetry which, in strong coupling limit, is shown to impart a unique flux tube structure to the QCD vacuum by generating a state of magnetic (dual) superconductivity. Flux tube energy computation has been used to analyze the QCD vacuum response in different energy sectors and the appearance of the confinement forces at large hadronic distances has been demonstrated. The analysis of the large scale behavior of QCD flux tube solutions has been used to compute the various confinement parameters in terms of different length and mass scales of dual QCD.

Keywords : Color confinement, Dual QCD, SU(2) and SU(3) symmetries

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1. Introduction

Among the four fundamental interactions of the nature, the strong interactions are believed to be generated by the non-Abelian $SU(3)_C$ gauge theory – the quantum chromodynamics – of colored quarks and gluons which are permanently confined in the color singlet hadronic bound states [1]. The dynamics of the hadron physics is believed to be completely described by QCD and the much more support for QCD derives from its unique ability to produce the asymptotic freedom property of quarks at short distances [2] which, in fact, explains the approximate scaling observed in deep inelastic scattering of leptons off hadrons. Though, the renormalized gauge coupling constant in perturbative QCD shows the asymptotic free property at large momentum, it becomes progressively large at small momentum (low energy sector) where the color confinement and chiral symmetry breaking are expected [3]. These two phenomena are quite outstanding features of the non-perturbative QCD. Since in QED, the strong coupling regime can be easily studied by going to the dual theory; one might attempt to understand the low energy sector of QCD within a dual theory. In the absence of a realistic dual theory of QCD, the dual QCD has been tried either phenomenologically [4] or through lattice gauge theories [5]. In order to understand the confinement mechanism, much attention has been paid during last few years for the analogy between superconductors and QCD vacuum [6-8]. The dual superconductivity of the QCD vacuum was advocated as the mechanism of the color confinement long ago primarily by 't Hooft [9] and Mandelstam [10] where the color confinement is brought by the dual Meissner effect originated from the colored monopole condensation. The color electric flux then seems to be excluded in QCD vacuum, which leads to the formation of the squeezed color electric flux tubes between colored sources. Recent lattice QCD simulation studies [11, 12] also show that the QCD monopole condensation plays a leading role in color confinement. In recent years, much of the efforts by various workers [13-15] in the area of establishing a suitable mechanism of confinement through monopole condensation in QCD vacuum has been motivated by the Abelian projection approach [16] in which the monopole appears as a topological object. It then seems that the idea of the monopole condensation in QCD vacuum is in essence justified. However, it is also true that very little is known about the dynamics that derives such phenomena. One, therefore, needs to develop some effective theory, which

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may incorporate the dynamics of the system for deriving the QCD magnetic condensation and may provide a dynamicsal basis for the explanation of the crucial confinement mechanism.

Based on such standpoint, a dual gauge theory out of SU(2) QCD has recently been constructed [17-20] which generates the monopole condensation on dynamical grounds and leads to a unique dual dynamics. In the present paper, the dynamical model for the QCD monopole condensation has further been investigated for the dual superconductivity of QCD and the associated implications. The dynamical structure of such dual gauge theory alongwith the nature of the confinement potential and its flux tube structure has been investigated. Starting from the review of the magnetic symmetry in QCD vacuum and the mathematical foundations of dual chromodynamics in fibre bundle form; the construction of the dual gauge potential in terms of magnetic vectors on global sections has been shown to lead the dual dynamics. The breaking of the magnetic symmetry in a dynamical way in strong coupling limit has been shown to impart a unique flux tube structure to the QCD vacuum by generating a state of magnetic superconductivity. The large scale behavior of the color electric flux tube solutions has been analyzed and the confinement parameters in terms of the different length and mass scales evolving as a result of the dynamical breaking of magnetic symmetry in dual QCD have been discussed. In the dynamically broken phase, the response of QCD vacuum in high and low energy sectors has been investigated through flux tube energy computation and the appearance of the confinement forces has been demonstrated at large hadronic distances. The numerical estimate of the confinement parameters in nonperturbative regime of QCD has been used to identify the nature of the superconductivity of the dual QCD vacuum.

2. Magnetic symmetry and duality in quantum chromodynamics

Before investigating the detailed dynamical structure of the dual QCD vaccum and the associated features, let us first briefly review the general formulation of dual QCD as a non-Abelian gauge theory with the magnetic symmetry [21]. In the higher-dimensional metric formulation of the gauge theory, one can visualize the gauge symmetry (G) as a n-dimensional isometry in the (4+n)-dimensional unified space P which leads to the identification of P as a principal fibre bundle $P(M, G)$ over space-time if the quotient space P/G is taken as the base manifold M with a canonical projection $\Pi : P \rightarrow M$. A connection on $P(M, G)$ admits a left isometry H which formally forms a subgroup of G (the right isometry) and commutes with it. With such considerations, one can introduce the magnetic symmetry to make the full QCD as a generalized gauge theory in terms of the following gauge covariant condition

$$D_\mu \hat{m} = 0, \tag{1}$$

which implies

$$\partial_\mu \hat{m} + g\, W_\mu \times \hat{m} = 0.$$

Here, $\hat{m}$ is a scalar multiplet belonging to the adjoint representation of the gauge group G whose little group is assumed to be a Cartan's subgroup at each space-time point Mathematically, it means that the magnetic symmetry restricts the connection to those whose holonomy bundle becomes a reduced bundle $P(M, H)$. For the simple choice of G and H as $SU(2)$ and $U(1)$ respectively, the exact solution of eq. (1) leads to the form of the potential given as

$$W_\mu = A_\mu \hat{m} - g^{-1} \hat{m} \times \partial_\mu \hat{m}, \tag{2}$$

where A_μ is the Abelian (color electric) component and the second part, determined completely by the magnetic symmetry, is of dual (color magnetic) topological in nature.

Thus, the topological structure may be brought into dynamics explicitly in a dual symmetric way by imposing the magnetic symmetry and the multiplet $\hat{m}$ may then be viewed to define the homotopy of the mapping $\Pi_2(S)^2$ on $\hat{m}$ $S_R^2 \rightarrow S^2 = SU(2)/U(1)$, where S_R^2 is the two-dimensional sphere of the three-dimensional space and S^2 is the group coset space fixed by $\hat{m}$. The dual symmetric structure of the formulation is reflected more clearly at the field strength level also where the associated generalized field strength may be identified as

$$G_{\mu\nu} = W_{\nu,\mu} - W_{\mu,\nu} + g\, W_\mu \times W_\nu \equiv \left(F_{\mu\nu} + B_{\mu\nu}^{(d)}\right)\hat{m}, \tag{3}$$

where

$$F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu} \text{ and } B_{\mu\nu}^{(d)} = -g^{-1}\, \hat{m} \cdot \left(\partial_\mu \hat{m} \times \partial_\nu \hat{m}\right). \tag{4}$$

However, the dual structure becomes more interesting when the topological structure is brought into dynamics explicitly. For this purpose, we choose a gauge and rotate $\hat{m}$ to a prefixed space-time independent direction (say ξ_3 in isospace) by a gauge transformation U as

$$\rightarrow \hat{\xi}_3 = [0,0,1]^T. \tag{5}$$

Using the parameterization for $\hat{m}$ as, $\hat{m}$ = $(\sin\alpha \cos\beta$ $\sin\alpha \sin\beta \cos\alpha)^T$ and choosing $U = \exp(-\alpha t_2 - \beta t_3)$ in accordance with eq. (3), we obtain the gauge potential W_μ in magnetic gauge as

$$W_\mu \xrightarrow{U} W'_\mu = \left(A_\mu + B_\mu\right)\hat{\xi}_3, \tag{6}$$

where B_μ is the magnetic potential, which in the magnetic gauge is given by

$$B_\mu = B_\mu \hat{m} = g^{-1} \cos\alpha\, \partial_\mu \beta\, \hat{m}. \tag{7}$$

In the magnetic gauge, the associated field strength $G_{\mu\nu}$, defined by eq. (3), is then obtained as

$$G_{\mu\nu} \xrightarrow{U} G'_{\mu\nu} = \left(F_{\mu\nu} + B^{(d)}_{\mu\nu}\right)\hat{\xi}_3, \tag{8}$$

where the part associated with the topological degree of freedom is expressible in the form of the dual potential as follows :

$$B^{(d)}_{\mu\nu} = B_{\nu,\mu} - B_{\mu,\nu}$$

$$= -g^{-1} \sin\alpha\left(\partial_\mu\alpha\,\partial_\nu\beta - \partial_\nu\alpha\,\partial_\mu\beta\right). \tag{9}$$

The potential B_μ here is identified as the magnetic potential associated with the topological monopole and is completely fixed by $\hat{m}$ upto the Abelian gauge degree of freedom. Hence, in the magnetic gauge, one may indeed bring the topological properties of $\hat{m}$ down to the dynamical variable B_μ by removing all non-essential degrees of freedom.

3. Dynamics of the dual QCD

In order to study the dynamics of the dual QCD and the associated mechanism responsible for the process of confinement, let us try to develop the field theoretical formulation for the dual QCD by using the potential given by eq. (6). The nontrivial subset of the original gauge theory for the simple case of $\underline{SU}$ (2) gauge group with a quark doublet source $\psi(x)$ may be derived from the Lagrangian given by ;

$$\pounds = -\frac{1}{4}G^2_{\mu\nu} + \bar{\psi}\, i\gamma^\mu D_\mu\psi - m_0\bar{\psi}\psi, \tag{10}$$

where $G_{\mu\nu}$ is the generalized gauge field strength corresponding to the constrained potential W_μ and is given by eq. (3). In the magnetic gauge, it leads to the dual symmetric field equations given in the following form,

$$G_{\mu\nu}{}^{,\nu} = F_{\mu\nu}{}^{,\nu} = j_\mu \text{ and } G^{(d)\,\nu}_{\mu\nu,} = B_{\mu\nu}{}^{,\nu} = k_\mu. \tag{11}$$

Such non-trivial dual structure has close relationship with the confining properties of QCD vacuum. However, in order to avoid the problems due to the pointlike structure and the singular behavior of the potential associated with monopoles, we may also use the regular dual magnetic potential $B^{(d)}_\mu$ (with corresponding field strength identified as $B_{\mu\nu}$) for the magnetic part and at the same time introduce a complex scalar field ϕ for the monopole. In addition, the confining properties of the QCD vacuum as a result of the non-trivial dual structure become more transparent if we express the Lagrangian in absence of the color electric sources. Under these considerations, let us reexpress the Lagrangian given by eq. (10) in the following form ;

$$\pounds^{(m)}_{dr} = -\frac{1}{4}B^2_{\mu\nu} + \left|\partial_\mu + i\frac{4\pi}{g}B^{(d)}_\mu\right|^2 - V(\phi), \tag{12}$$

where, the effective value of the potential $V(\phi)$ is obtained by using the single-loop expansion technique [22] alongwith the requirements of ultraviolet finiteness and infrared instability of the dual QCD Lagrangian and is given by ;

$$V(\phi) = 24\pi^2 \quad \phi_0^4 + (\phi^*\phi)^2 \Big|\, 2\ln \quad -1 \tag{13}$$

$(\phi_0 = \langle\phi^*\phi\rangle^{\frac{1}{2}}$ being the vacuum expectation value of ϕ). The lagrangian given by eq. (12) generates the dynamical breaking of magnetic symmetry due to the effective potential and forces the magnetic condensation of the QCD vacuum. This is precisely what one needs as it guarantees the appearance of the dual Meissner effect which confines any colored flux. Since the above Lagrangian resembles with that of Ginzburg-Landau for superconductivity, the magnetic condensation of QCD vacuum leads to a definite flux tube structure to the dual QCD vacuum imparting it to the appropriate (color) electric flux confining properties. It is, therefore, naturally desired to analyze the flux tube structure and the nature of the magnetically condensed vacuum. For this purpose, let us try to analyze the field equations associated with the Lagrangian given by eq. (12) which are derived in the following form ;

$$\partial^\mu \; -i\frac{4\pi}{g}B^{(d)\mu} \quad \partial_\mu + i\frac{4\pi}{g}B^{(d)}_\mu$$

$$\cdot\frac{24\pi^2}{g^4}\left(4\phi\phi^*\ln\frac{\phi\phi^*}{\phi_0^2}\right|\phi = 0, \tag{14}$$

$$\partial^\nu B_{\mu\nu} + i\frac{4\pi}{g}\left(\phi^*\overleftrightarrow{\partial}_\mu\phi\right) - \frac{32\pi^2}{g^2}B^{(d)}_\mu\,\phi\phi^* = 0. \tag{15}$$

We focus on the single flux tube solution using the cylindrical symmetry (ρ,φ,z) and the flux tube orientation along the z-axis. For such system, the dual gauge field and the monopole field can be expressed as

$$B^{(d)}_\mu = g^{-1}\cos\alpha\;\partial_\mu\beta,$$

and $\quad \phi(x) = \exp(i\,n\,\varphi)\,\chi(\rho), \quad (n = 0, \pm 1, \pm 2, \dots). \qquad (16)$

It leads to the following cylindrical components for the dual gauge field,

$$B^{(d)}_\rho = \frac{1}{\,}\cos\alpha\,\partial_\rho\beta,$$

$$B^{(d)}_\varphi = \frac{1}{\,}\cos\alpha\frac{1}{\,}\partial_\varphi\beta, \tag{17}$$

$$B^{(d)}_z = \frac{1}{\,}\cos\alpha\,\partial_z\beta.$$

In view of the uniqueness of the function $\phi(x)$, we have

$$B_{\varphi}^{(d)}(x) = B(\rho), \quad \text{as} \quad B_t^{(d)} = B_\rho^{(d)} = B_z^{(d)} = 0. \quad (18)$$

Let us now take E_m with z-component and B with φ-component only. It leads to

$$B = B(\rho) = \frac{1}{g}\cos\alpha \frac{1}{\rho}\partial_\varphi \beta, \quad (19)$$

and the color electric field has the form as given below :

$$E_m(\rho) = -\frac{1}{\rho}\frac{d}{d\rho}[\rho B(\rho)]. \quad (20)$$

Using cylindrical form of the potentials given by eq. (16 and 17), the field equations can be derived in the following forms :

$$\frac{d}{d\rho}\left[\frac{1}{\rho}\frac{d}{d\rho}\left(\frac{1}{g}\cos\alpha\,\partial_\varphi\beta\right.\right.$$

$$+\frac{8\pi}{g}\left(\frac{n}{\rho}+\frac{4\pi}{g^2\rho}\cos\alpha\,\partial_\varphi\beta\right)\chi^2 = 0, \quad (21)$$

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{d\chi}{d\rho}\right) - \left|\left(\frac{n}{\rho}+\frac{4\pi}{g^2\rho}\cos\alpha\,\partial_\varphi\beta\right.\right.$$

$$+\frac{24\pi^2}{g^4}\left(4\chi^2\ln\frac{\chi^2}{\phi_0^2}\right.\quad \chi = 0. \quad (22)$$

Using eq. (19), these equations can be reduced to their simplest form as

$$\frac{d}{d\rho}\left[\frac{1}{\rho}\frac{d}{d\rho}(\rho B(\rho))\right] - \frac{8\pi}{g}\left(\frac{n}{\rho}+\frac{4\pi}{g}B(\rho)\right)\chi^2 = 0, \quad (23)$$

$$\frac{1}{\rho}\frac{d}{d\rho}\left(\rho\frac{d\chi}{d\rho}\right) - \quad \frac{n}{\rho}+\frac{4\pi}{g}B(\rho)$$

$$+\frac{24\pi^2}{g^4}\left(4\chi^2\ln\frac{\chi^2}{\phi_0^2}\right.\quad \chi = 0. \quad (24)$$

These are the required field equations, which govern the complete flux tube structure of the dual QCD vacuum when the dynamical breakdown of the magnetic symmetry takes place. We shall try to find the analytic solutions of these field equations and analyze their large-scale behavior mainly to explore the confining properties of the QCD vacuum in next section.

4. Flux tube structure and the confinement parameters in dual QCD

Let us evaluate the static finite energy for the flux-tube configuration governed by the field eqs (23) and (24). Using the Lagrangian density given by eq. (12), the Hamiltonian in the temporal gauge ($B_0 = 0$) is obtained in the form given below :

$$H = \int dz\, K\{B, \chi\},$$

where

$$K(B,\chi) = 2\pi\int^{\infty}\rho\,d\rho\left[\frac{}{2\rho^2}\left(\frac{d}{d\rho}(\rho B(\rho))\right)^2 + \left(\frac{d\chi}{d\rho}\right.\right.$$

$$+\left(\frac{4\pi}{g}B(\rho)+\frac{n}{\rho}\right)^2\chi^2 + \frac{24\pi^2}{g^4}\left(\phi_0^4 + (\chi)^4\left(2\ln\frac{\chi^2}{\phi_0^2}-1\right)\right) \quad (25)$$

The simplest solution that minimizes the energy of the configuration in dual QCD vacuum is obtained as

$$B(\rho) = -\frac{ng}{4\pi\rho} \quad \text{and} \quad \chi = \phi_0, \quad (26)$$

which is referred as the classical vacuum solution satisfying the equation of motion. In order to get other general solutions of the coupled nonlinear differential eqs. (23) and (24), let us impose the boundary condition that in three-dimensional case, there exists a choice of gauge in which

$$B(\rho,\varphi) \rightarrow -\hat{\varphi}\frac{ng}{4\pi\rho} \quad \text{and} \quad \rightarrow \phi_0. \quad (27)$$

These are the boundary conditions for the scaled fields, which lead to a unique solution of the equations of motion as ρ approaches to infinity. These are simply the results of the requirement requiring that ϕ approaches to its physical vacuum value in the superconductor and that $B(\rho)$ vanishes at large distances from the monopole-antimonopole pair. However, in the absence of any exact solution to the coupled differential equations, the asymptotic solutions can be obtained if we take the variation for $B(\rho)$ to get the appropriate asymptotic behavior of dual gauge potential as

$$B(\rho) = -\frac{ng}{4\pi\rho}[1 + F(\rho)]. \quad (28)$$

On using such variation in the field equations and under large ρ considerations, the function $F(\rho)$ for correct asymptotic behavior is given by

$$F(\rho) \xrightarrow[\rho\to\infty]{} C\rho^{\frac{1}{2}}\exp\left(-\frac{4\pi}{g}\sqrt{2}\phi_0\rho\right),$$

(*C* being a constant). (29)

The energy per string length for large ρ considerations is then obtained as

$$K = 2\pi \int_0^\infty \rho \, d\rho \left[\frac{n^2 g^2}{32\pi^2 \rho^2} (F'(\rho))^2 + \frac{n^2}{\rho^2} F^2(\rho) \chi^2 \right.$$

$$\left. + (\chi')^2 + \frac{24\pi^2}{g^4} \left(\phi_0^4 + \chi^4 \left| 2 \ln \frac{\chi^2}{\phi_0^2} - 1 \right. \right) \right] \tag{30}$$

(where the prime with function $F(\rho)$ and χ denotes their ρ-differentials).

This expression alongwith the solution given by eq. (29) and the energy minimization in the asymptotic limit, indicates the condensation of monopoles in dual QCD vacuum which results in the color electric flux confinement ultimately. Thus, in the dynamically broken phase of the magnetically condensed dual QCD vacuum, the penetration depth for the color electric flux is obtained as

$$\lambda_{QCD}^{(D)} = \left(4\pi\sqrt{2} \, g^{-1} \phi_0 \right)^{-1} \tag{31}$$

The squeezing of the color electric flux and the resulting color confinement in dual QCD vacuum can be visualized more clearly on energetic ground if we introduce a new variable $\rho = R \sin\theta$ (R, θ, and φ being the polar coordinates). The single (color) flux tube solution governed by the eqs. (23) and (24) then corresponds to the large R limit ($R \to \infty$) such that $R >> \rho$ alongwith the extremely small $\theta \, (\theta \to 0)$. Under these considerations, the energy expression for the infinitely long flux tube structure in dual QCD, given by eq. (30), can be rederived in the following form :

$$K = K_C \, R^2 + K_D \frac{1}{R^2} + K_0 \, , \tag{32}$$

where the functions K_C, K_D and K_0 are given in the following integral forms

$$K_C = \frac{24\pi^3}{g^4} \int_0^\pi \left\{ \phi_0^4 + \chi^4 \left(2 \ln \frac{\chi^2}{\phi_0^2} - 1 \right) \right\} \sin 2\theta \, d\theta , \tag{33}$$

$$\left(\frac{n^2 g^2}{8\pi} \right) \int_0^\pi \left(\frac{\partial F}{\partial \theta} \right) \left(\frac{}{\sin 2\theta} \right) d\theta \tag{34}$$

and

$$= 2\pi \int_0^\pi \left(\frac{\partial \chi}{\partial \theta} \right)^2 \tan\theta \, d\theta + 2\pi \, n^2 \int_0^\pi F^2 \chi^2 \cot\theta \, d\theta . \tag{35}$$

The energy expression given by eq. (32) now gives clearer picture for its implications on the confinement-deconfinement phase structure of the dual QCD vacuum. The various terms, in fact, signify the large-scale and small-scale behavior of the QCD vacuum (in the sense of the hadronic distances). For the sufficiently large hadronic distances, the first term of eq. (32) dominates. The R-independent coefficient associated with this term, given by eq. (33), shows that the corresponding energy gets minimized when the monopole field (χ) picks up its non-zero VEV (ϕ_0). It signals the appearance of an order parameter ϕ_0 which indicates the breaking of the magnetic symmetry of QCD vacuum in a dynamical way and leads the condensation of QCD monopoles. The color electric field then gets localized around the thin flux tube extending from $\theta = 0$ to $\theta = \pi$ which ultimately pushes the whole QCD vacuum to the confining phase. On the other hand, the small scale behavior of the system is entirely governed by the second term of the eq. (32) which, in fact, is not associated with any order parameter and is not expected to promote any magnetic condensation in dual QCD vacuum and therefore ultimately pushes the system to the deconfined phase. Furthermore, in the confining phase, the dynamical breaking of the magnetic symmetry in QCD vacuum leads to the dual Meissner effect and sets two characteristic mass scales. One is specified by the dual gauge mass (m_B) and identifies the magnitude of dual Meissner effect. The other is the scalar mode (m_ϕ), which corresponds to the energy threshold for monopole excitation in QCD vacuum and determines the rate of magnetic condensation around a colored source in QCD vacuum. These two scales are basically related to the penetration depth $\left(\lambda_{QCD}^{(D)}\right)$ and coherence length $\left(\xi_{QCD}^{(D)}\right)$ in the following manner :

$$m_B = \left(\lambda_{QCD}^{(D)} \right)^{-1} \quad \text{and} \quad \xi_{QCD}^{(D)} = \left(m_\phi \right)^{-1} . \tag{36}$$

The ratio of these two scales, as fixed by the effective potential (13), is given by

$$m_\phi / m_B = \sqrt{3} (2\pi\alpha_s)^{-1/2} \tag{37}$$

where $\alpha_s (= g^2/4\pi)$ is the strong coupling constant. In order to precisely understand the nature of the dual QCD vacuum, the numerical computation of the characteristic length and mass scales plays a major role. Such computation involves two free parameters; one is the g which is related to α_s and the other is (ϕ_0) which is associated with the string tension and the Regge slope parameter. Keeping in mind the running nature of the strong coupling constant in QCD, the experimentally confirmed values of α_s happen to lie in the range given by $0.1 < \alpha_s < 0.2$ for the energy region $\Lambda_{QCD} >> 1$ GeV, where the perturbative QCD works well [23]. However, in the low energy region ($\Lambda_{QCD} < 1$ GeV), the strong coupling rises appreciably beyond the value

$(0.2 < \alpha_S < 1)$ and pushes the QCD vacuum to the non-perturbative phase [24]. Hence, considering the value of $\alpha_S \approx 0.22$ as the minimum value of the strong coupling in non perturbative regime, we obtain the corresponding value of g as, $g = 1.66$. Further, in dual QCD, since the string tension is the energy per unit length carried by the flux tube, we have

$$\alpha' = (2\pi K)^{-1} = (2\pi\gamma\phi_0^2)^{-1} \tag{38}$$

with $\alpha' = 0.93\ \text{GeV}^{-2}$ as the Regge slope parameter. Using the energy expression given by eq. (30), it leads to the ϕ_0-value, as $|\phi_0| \approx 0.156$ GeV. Using these data alongwith the eqs. (36) and (37) for the typical values of $\alpha_S \approx 0.22$, we obtain the numerical estimate for the characteristic length scales and mass scales in dual QCD as follows :

$$\lambda_{QCD}^{(D)} = 0.12 \text{ fermi}, \qquad m_B = 1.66\,\text{GeV},$$
$$\xi_{QCD}^{(D)} = 0.08 \text{ fermi}, \qquad m_\phi = 2.44\,\text{GeV}. \tag{39}$$

One can now define the Ginzburg-Landau parameter for the dual QCD vacuum as the ratio given by $\kappa_{QCD}^{(D)} = \lambda_{QCD}^{(D)} / \xi_{QCD}^{(D)}$ which for the present case, yields the value $\kappa_{QCD}^{(D)} \approx 3/2$. Since $\kappa_{QCD}^{(D)} > 1$, it demonstrates that the magnetically condensed QCD vacuum with the above discussed flux tube structure acquires the type-II superconducting state for the near physically realizable strong couplings in non-perturbative regime of the QCD.

5. Conclusions

The gauge potential constructed in terms of the magnetic vectors on global sections using the magnetic symmetry has been shown to describe the dual dynamics associated with non-Abelian monopoles in dual QCD. The dual magnetic potential, derived in terms of eq. (17) in magnetic gauge, is of completely topological in origin. In the low energy sector of QCD where the perturbative techniques lose their meaning, the dynamics of the system has been shown to play an important role in various typical non-perturbative effects. The dynamical breaking of the magnetic symmetry, leading to the state of the magnetic condensation, has been shown to impart a unique flux tube structure to the dual QCD vacuum which in turn, generates the appropriate color confining properties. The field equations of dual QCD derived in terms of eqs. (14) and (15), when solved in strong coupling limit, allow the finite energy flux tube solutions which appear as a dual version of Abriksov vortices. The analysis of the large-scale (low energy) behavior of these solutions yields one of the characteristic length scales of dual QCD, *viz* the penetration length for color electric field. The other scale is then set by the effective penetration length for color electric field. The other scale is then set by the effective potential responsible for the dynamical symmetry breaking and the strong coupling constant of dual QCD. For the numerical computation of these characteristic scales of dual QCD, the energy expression of flux tube is derived in terms of eq. (30) which may be used to identify the nature of different terms when one introduces the flux tube in the hadronic sphere going through the two poles located at $\theta = 0$ to $\theta = \pi$. Energy expression (32) rederived from that given by eq. (30), in fact, does exactly this job and clearly identifies the response of QCD vacuum in high and low energy sectors. In the low energy sector at large hadronic distances, it leads to the appearance of the confinement forces in QCD vacuum while the deconfinement phase is indicated at small scale (high-energy sector). The numerical computation of the characteristic mass and length scales for a particular coupling in the non-perturbative sector, as undertaken in the last part, helps to understand the nature of dual QCD vacuum. The estimate for the GL-parameter in non-perturbative regime with $\alpha_S \approx 0.22$, demonstrates the type-II superconducting behavior of the QCD vacuum, which is in agreement with the recent results of Koma *et al* [25] and Maedan *et al* [26]. The treatment may be used to further investigate the phase transition scale where the QCD monopoles disappear and the system switches over to the normal deconfined phase from its flux tube phase. It may be very useful for the study of QGP (quark-gluon plasma) formation in terms of the flux tubes in dual QCD.

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